SCATTERING OF AN EM WAVE OFF A WHISTLER WAVE IN THE IONOSPHERE

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Scattering of radio waves from the ionosphere has been a subject of extensive investigation [1–2], in the past and is powerful tool for the plasma diagnostics. An obliquely propagating whistler wave of frequency ω, with respect to ambient magnetic field in the ionosphere, disturbs the electron density of the plasma. When a high frequency electromagnetic (EM) wave of frequency ω_0 is sent into the disturbed region, the EM wave gets scattered with a frequency shift ω. In this communication, we study the scattering of radio waves from the F–region of the ionosphere. We are considering linearly polarized waves, hence the region of validity is below the x–mode reflection layer. However, the preformed duct that we are considering here can occur at any height. We choose the pump frequency ω_0 to have turning point well above the whistler duct. The pump wave can cause density striations near the upper hybrid layer by the superposition of the upward and downward going waves. However we are not considering these effects. Using the wave equation for the propagation of the whistler wave [3] through the ionospheric plasma in conjunction with the fluid equations, an expression for the perturbed electron density is obtained. The density perturbation couples with the oscillatory electron velocity due to the high frequency incident wave to produce a nonlinear current density. Wave equation has been solved to obtain the expression for the ratio (P_r) of the scattered power to that of the incident power. Numerical calculations have been made for the ratio P_r for equatorial F–region, for Northern Scandinavian and at Arecibo.

Let us consider the propagation of the whistler wave through the F–region of the ionosphere with ambient magnetic field $\mathbf{B}_s$ making an angle of α with the vertical X–axis and a wave vector $\mathbf{k}$ lying in the $x-z$ plane,

$$\mathbf{E} = \mathbf{A}\exp[i(\omega t - k_x x - k_z z)] \tag{1}$$

where $k = \omega\omega_{pe}/c\sqrt{(\omega\omega_{ce}\cos\theta)}$, ω_{pe} and ω_{ce} are the electron plasma and electron cyclotron frequencies and θ is the angle between $\mathbf{k}$ and $\mathbf{B}_s$ and c is the velocity of light. The perturbed electron density due to the propagation of whistler wave can be written as

$$n = -\mathbf{k}.(\sigma.\mathbf{E})/e\omega, \tag{2}$$

where σ is conductivity tensor having different components.

Let a high frequency EM wave of frequency $\omega_0 \gg \omega_{ce}$ with its electric vector

$$\mathbf{E}_0 = \hat{y}A_0\exp[i(\omega_0 t - k_{0x}x - k_{0z}z)] \tag{3}$$

propagate through the plasma perturbed by the whistler wave. The oscillatory velocity $\mathbf{V}_0 = -e\mathbf{E}_0/\mathrm{i}m\omega_0$ due to the EM wave in conjunction with the density perturbation n produces a current of frequency $\omega_0 + \omega = \omega_1$, given by

$$\mathbf{J}_1 = -1/2en\mathbf{V}_0 - en_0\mathbf{V}_1 \tag{4}$$

where second term is due to the self consistent field $\mathbf{E}_1$ of the scattered wave of frequency ω_1. Phase matching conditions for this there wave resonant scattering process are given by

$$\omega_1 = \omega_0 + \omega \quad \text{and} \quad \mathbf{k}_1 = \mathbf{k}_0 + \mathbf{k} \tag{5}$$

and linear dispersion relations for the incident EM wave and scattered wave are as follows

$$k_0^2c^2 = \omega_0^2 - \omega_{\mathrm{pe}}^2 \quad \text{and} \quad k_1^2c^2 = \omega_1^2 - \omega_{\mathrm{pe}}^2. \tag{6}$$

Using (5) and (6), the phase matching conditions yield

$$\mathbf{k}.(\mathbf{k} + 2\mathbf{k}_0) = 0. \tag{7}$$

Equation (7) is satisfied in two case: 1) $\mathbf{k} + 2\mathbf{k}_0 = 0$ or 2) $\mathbf{k} \perp \mathbf{k} + 2\mathbf{k}_0$. The condition 2) is ruled out for low frequency whistlers.

Taking the electric field of the scattered wave to be

$$\mathbf{E}_1 = \hat{y}A_1(x,z)\exp[\mathrm{i}(\omega_1 t - k_{1x}x - k_{1z}z)] \tag{8}$$

and substituting it in the wave equation, we get

$$\frac{\partial^2 A_1}{\partial x^2} - 2\mathrm{i}k_{1x}\frac{\partial A_1}{\partial x} + \frac{\partial^2 A_2}{\partial z^2} - 2\mathrm{i}k_{1z}\frac{\partial A_1}{\partial z} = \frac{4\pi\mathrm{i}\omega_1}{c^2}J_{1y}^{NL}. \tag{9}$$

Introducing two new variables ξ, η along and perpendicular to wave vector $\mathbf{K}_0$ of EM wave respectively and using-proper transformation equation and WKB approximation above equation can be simplified to

$$\frac{\partial A_1}{\partial \xi} = F\mathrm{A}_y\mathrm{A}_{0y}\exp\left(-\frac{\xi^2}{\xi_0^2}\right) \tag{10}$$

where ξ_0 and A_y are the finite width and amplitude of the whistler wave respectively and A_{0y} is the amplitude of the EM wave and F is given by

$$F = \frac{4\pi^2 e\omega_1\sigma_{yy0}k}{c^2 m_e\omega\omega_{\mathrm{pe}}^2 k_1}[Q_1\cos(\alpha-\theta) + Q_2\sin(\alpha-\theta)]. \tag{11}$$

Where Q_1, Q_2 are the functions of the conductivity tensor components. On integration (10) from $\xi = -\infty$ to $+\infty$, we get

$$\frac{A_1}{A_{0y}} = \sqrt{\pi}F\xi_0 A_y \tag{12}$$

and the ratio of the power of the scattered wave to incident EM wave is given by

$$P_r = \left|\frac{A_1}{A_{0y}}\right|^2. \tag{13}$$

For Northern Scandinavian ($\alpha = 13°$) and at Arecibo ($\alpha = 40°$) for $\theta = 10°$ EM wave frequency $\omega_0/2\pi = 10$ MHz, whistler wave frequency $\omega/2\pi = 3$ kHz for 1 MW incident power of EM wave the power of scattered wave would be about 40 μW and 3 mW respectively and for equatorial region ($\alpha = 90°$) the scattered power would be about 3 μW for the above parameters.

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