ELECTROMAGNETIC EMISSION IN A MAGNETIZED PLASMA

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We have presented a theory of emission of electromagnetic waves, in the ordinary and extraordinary modes, in the presence of Langmuir turbulence in a magnetized plasma due to the high-frequency nonlinear force. The mechanism of emission considered is the plasma-maser interaction which is essentially an energy up-conversion process. The growth rates of the ordinary and extra-ordinary mode emissions are calculated and the results are compared with those obtained from the direct formulation. The scope of application of the results to radio spectra from solar flares is then stressed.

1. INTRODUCTION

According to the recent weak turbulence theory, the lowest-order mode-mode coupling processes are composed of three parts: the nonlinear scattering [1], the resonant three-wave interaction [2], and the plasma maser [3]. The thrid process, viz., the plasma-maser interaction, which was first pointed out independently by Nambu [4] and Tsytovich et al. [5], is basically an energy up-conversion process in which energy flows from a low-frequency resonant mode to a high-frequency nonresonant radiation field via the resonant electrons. However, recent works have also demonstrated the possibility of causing transfer of energy from a high-frequency resonant field to a low-frequency nonresonant field as well [6].

Furthermore, the nonlinear effect of the resonant and nonresonant modes simultaneously on the evolution of the electron distribution function is called the inverse plasma maser [7]. Quite recently, it is shown that total energy, momentum conservation relations between particle kinetic energy and the wave energy are exactly satisfied for the plasma-maser process [8]. The Manley-Rowe relation [9] for plasma waves is violated [8] and as a result an efficient energy up-conversion from the low-frequency mode to the high-frequency mode is possible even for a normal unreversed electron population [10].

The plasma-maser effect, which is effective both for Landau and cyclotron resonance and which coexists with the linear Landau damping, has attracted in the past several years considerable interest and critical comments regarding its efficiency and limitations as an up-conversion mechanism [11-12]. The plasma-maser processes both for the nonresonant mode, the resonant mode and for the particle (inverse plasma maser) are summarized in Table. Here, D and P indicate the contribution from the direct and polarization terms, respectively. It should be mentioned that the polarization contribution cannot be neglected, and the polarization contribution very often gives the dominant plasma–maser effect even for unmagnetized plasma [8, 10]. Unfortunately, the recent review article [13] completely neglects the importance of the polarization contributions in the plasma–maser physics.

Table

Plasma–maser interaction

	Unmagnetized plasma		Magnetized plasma	
	D	*P*	*D*	*P*
Nonresonant mode	REF.13	no contribution	REF.4	REF.22
Resonant mode	REF.13	REF.10	REF.11	REF.11
Particle	REF.13	REF.8	not studied	not studied

It is now well established that the plasma maser effect does not exist as an internal nonlinear process for isotropic and homogeneous plasma turbulence [12, 13]. Because the plasma maser contribution exactly cancells out with the reverse absorption process which comes from the quasilinear interaction between the resonant mode and electrons [8, 12, 13]. In other words, the plasma maser effect works only for plasmas with symmetry–breaking factors, such as anisotropy due to external magnetic field [14], inhomogeneity of plasmas, etc.

It has been seen that the plasma–maser effect can be best understood in terms of a high–frequency nonlinear force [3], which arises as a result of the resonant interaction between electrons and modulated fields caused by cupling between a test high–frequency nonresonant wave field and a resonant turbulent field of the system. This high–frequency nonlinear dissipative force accelerates or decelerates the electrons, and the accelerated electrons can then emit electrostatic or electromagnetic waves. Unlike the ponderomotive force in the parametric interaction [2], this nonlinear force is a high–frequency one and makes the nonresonant high–frequency wave unstable.

In the present paper, we extend the earlier work of generation of electromagnetic waves in an unmagnetized plasma [15] and consider here the emission of electromagnetic waves, in the ordinary and extra–ordinary modes, in a magnetized plasma in the presence of Langmuir turbulence from the nonlinear force consideration. In Sec.2, we obtain an expression for the nonlinear force discussed above. The growth rates of ordinary and extraordinary modes are calculated in Sec.3, and 4, respectively. The application of the results to anomalous radiation from solar flares is discussed in Sec.5. Finally, Sec.6 contains the discussion of our work.

2. FORMULATION

We consider a homogeneous magnetized plasma with the applied magnetic field $\vec{B}_0 = \hat{Z} B_0$ in the presence of an enhanced stationary Langmuir turbulence caused by an electron beam drifting through the plasma. The basic equations governing the interaction of a test electromagnetic wave with the stationary Langmuir turbulence are the Vlasov equations for the electrons and the Maxwell equations:

$$\left\{\frac{\partial}{\partial t} + \vec{v}\cdot\frac{\partial}{\partial \vec{r}} - \frac{e}{m}\left(\vec{E}(\vec{r},t) + \frac{\vec{v}\times\vec{B}(\vec{r},t)}{c}\right)\cdot\frac{\partial}{\partial \vec{v}}\right\} f_e(\vec{r},\vec{v},t) = 0, \quad (1)$$

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad (2)$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}, \quad (3)$$

$$\vec{J} = -e\int \vec{v}\, f_e(\vec{r},\vec{v},t)\,\mathrm{d}\vec{v}, \quad (4)$$

where $f_e(\vec{r},\vec{v},t)$ is the electron distribution function and the other notations are standard.

The total fields and electron distribution function can de written as

$$\vec{E} = \varepsilon\,\vec{E}_\ell + \mu\,\delta\vec{E}_h + \mu\varepsilon\,\delta\vec{E}_{\ell h}, \quad (5)$$

$$\vec{B} = \vec{B}_0 + \mu\,\delta\vec{B}_h + \mu\varepsilon\,\delta\vec{B}_{\ell h}, \quad (6)$$

$$f_e = f_{0e} + \varepsilon f_{1e} + \varepsilon^2 f_{2e} + \mu\delta f_h + \mu\varepsilon\delta f_{\ell h} + \mu\varepsilon^2\Delta f. \quad (7)$$

Here, f_{0e} is the space– and time–averaged part of the electron distribution function, and f_{1e} and f_{2e} are the fluctuating parts. ε is a small parameter representing the strength of the turbulent field $\vec{E}_\ell$. $\mu\,\delta\vec{E}_h$ and $\mu\,\delta\vec{B}_h$ are the electric and magnetic fields of the test electromagnetic waves, and $\mu\,\delta f_h$ is the corresponding perturbed electron distribution function. $\delta\vec{E}_{\ell h}$ and $\delta\vec{B}_{\ell h}$ are the modulation fields while $\delta f_{\ell h}$ and Δf are the corresponding electron distribution functions. We assume that $\mu \ll \varepsilon$. Here, we have omitted the $\mu\varepsilon^2\,\Delta\vec{E}$ and $\mu\varepsilon^2\,\Delta\vec{B}$ fields. Such a step is justified under the random phase approximation adopted in the present analysis [16].

The nonlinear dissipative force can be defined as the rate of change of momentum give by, to the order $\mu\varepsilon^2$

$$\frac{\partial}{\partial t}\int m\vec{v}\Delta f\,d\vec{v} = \frac{\partial}{\partial t}\int m\vec{v}\sum_{K,\,\Omega}\Delta f(\vec{K},\,\Omega)\times\exp\left[i(\vec{K}\cdot\vec{r}-\Omega t)\right]d\vec{v}, \qquad (8)$$

where $(\vec{K},\,\Omega)$ correspond to the wave vector and the frequency of the electromagnetic waves. Therefore, the Fourier component of the high-frequency nonlinear dissipative force acting on unit volume of electrons is given by

$$\vec{F}_{Nh}(\vec{K},\,\Omega) = -i\,\Omega\,m\,n_b\int\vec{v}\,\Delta f(\vec{K},\,\Omega)\,d\vec{v}, \qquad (9)$$

where n_b is the number density of background electrons.

3. ORDINARY MODE INSTABILITY

We now consider that the test electromagnetic wave under consideration is in the ordinary mode with the wave vector $\vec{K} = (K_\perp,\,0,\,0)$ propagating along x-direction. The linearized high-frequency electron equation of motion, with the nonlinear dissipative force term as given by Eq. (9) can be written as

$$mn_b\,\frac{\partial\vec{v}_h}{\partial t} = -en_b\,\delta\,\vec{E}_h + \vec{F}_{Nh}. \qquad (10)$$

With the help of Maxwell equations together with Eq. (10), we finally obtain, following the usual procedure, the dispersion relation of the O-mode in the presence of the nonlinear dissipative force as

$$1-(c^2K_\perp^2+\omega_{pe}^2)/\Omega^2 = -(4\pi ief_h/m\Omega^2). \qquad (11)$$

where

$$f_h = F_{Nhz}(K,\,\Omega)/i\delta E_h(K,\,\Omega). \qquad (12)$$

The LHS of Eq.(11) represents the linear dielectric function $\varepsilon_0(\vec{K},\,\Omega)$ of the O-mode. We can, therefore, write Eq. (11) as

$$\varepsilon_0(\vec{K},\,\Omega) = -\frac{4\pi ie}{m\Omega^2}f_h. \qquad (13)$$

We now write $\Omega = \Omega_r + i\gamma$, where Ω_r is the real frequency of the O-mode, and γ is its growth rate. We then get

$$\varepsilon_0(\vec{K},\,\Omega_r) = 0, \qquad (14)$$

and

$$\gamma = -\frac{4\pi ief_h}{m\Omega^2}\left(\frac{\partial\varepsilon_0}{\partial\Omega}\right)^{-1}. \qquad (15)$$

Substituting for Δf [16] in Eq. (9), we obtain

$$F_{NhZ}(\vec{K},\,\Omega) = \Omega m n_b \left(\frac{e}{m}\right)^3 \delta E_h(\vec{K},\,\Omega) \sum_{\vec{K},\omega} |E_\ell(\vec{k},\,\omega)|^2 \times$$

$$\times \left[(P+Q) - \frac{\omega_{pe}^2(\Omega-\omega)(A+B)\times(C+D)}{R(\vec{K}-\vec{k})\{(\Omega-\omega)^2 - c^2K_\perp^2\}}\right], \quad (16)$$

where $\vec{k}$ and ω are respectively the wave vector and the frequency of the Langmuir wave, and

$$P = \sum_{s,a,n=-\infty}^{\infty} \int v_\parallel \frac{J_s^2(K_\perp v_\perp/\Omega_e)}{s\Omega_e - \Omega} \frac{\partial}{\partial v_\parallel} \frac{J_a^2(K_\perp v_\perp/\Omega_e)}{a\Omega_e - k_\parallel v_\parallel - (\Omega-\omega)} \times$$

$$\times \frac{\partial}{\partial v_\parallel} \frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega - n\Omega_e} \left\{\left(1 - \frac{n\Omega_e}{\Omega}\right)\frac{\partial}{\partial v_\parallel} + \frac{n\Omega_e v_\parallel}{\Omega v_\perp}\frac{\partial}{\partial v_\perp}\right\} f_{0e}\,d\vec{v}, \quad (17)$$

$$Q = \sum_{s,a=-\infty}^{\infty} \int v_\parallel \frac{J_s^2(K_\perp v_\perp/\Omega_e)}{s\Omega_e - \Omega} \frac{\partial}{\partial v_\parallel} \frac{J_a^2(K_\perp v_\perp/\Omega_e)}{a\Omega_e - k_\parallel v_\parallel - (\Omega-\omega)} \times$$

$$\times \left\{\left(1 - \frac{a\Omega_e}{\Omega}\right)\frac{\partial}{\partial v_\parallel} + \frac{a\Omega_e v_\parallel}{\Omega v_\perp}\frac{\partial}{\partial v_\perp}\right\} \frac{1}{\omega - k_\parallel v_\parallel - i0} \frac{\partial}{\partial v_\parallel} f_{0e}\,d\vec{v}, \quad (18)$$

$$A = \sum_{s,a=-\infty}^{\infty} \int v_\parallel \frac{J_s^2(K_\perp v_\perp/\Omega_e)}{s\Omega_e - \Omega} \frac{\partial}{\partial v_\parallel} \frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega - \omega + k_\parallel v_\parallel - a\Omega_e} \times$$

$$\times \left\{\left(1 - \frac{a\Omega_e}{\Omega-\omega}\right)\frac{\partial}{\partial v_\parallel} + \frac{a\Omega_e v_\parallel}{(\Omega-\omega) v_\perp}\frac{\partial}{\partial v_\perp}\right\} f_{0e}\,d\vec{v}, \quad (19)$$

$$B = \sum_{s=-\infty}^{\infty} \int v_\parallel \frac{J_s^2(K_\perp v_\perp/\Omega_e)}{s\Omega_e - \Omega} \left\{\left(1 - \frac{s\Omega_e}{\Omega-\omega}\right)\frac{\partial}{\partial v_\parallel} +\right.$$

$$\left. + \frac{s\Omega_e v_\parallel}{(\Omega-\omega) v_\perp}\frac{\partial}{\partial v_\perp}\right\} \frac{1}{\omega - k_\parallel v_\parallel + i0} \frac{\partial}{\partial v_\parallel} f_{0e}\,d\vec{v}, \quad (20)$$

$$C = \sum_{a,n=-\infty}^{\infty} \int v_\parallel \frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega - \omega + k_\parallel v_\parallel - a\Omega_e} \frac{\partial}{\partial v_\parallel} \frac{J_n^2(K_\perp v_\perp/\Omega_e)}{\Omega - n\Omega_e} \times$$

$$\times \left\{\left(1 - \frac{n\Omega_e}{\Omega}\right)\frac{\partial}{\partial v_\parallel} + \frac{n\Omega_e v_\parallel}{\Omega v_\perp}\frac{\partial}{\partial v_\perp}\right\} f_{0e}\,d\vec{v}, \quad (21)$$

$$D = \sum_{a=-\infty}^{\infty} \int v_\parallel \frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega - \omega + k_\parallel v_\parallel - a\Omega_e} \left\{\left(1 - \frac{a\Omega_e}{\Omega}\right)\frac{\partial}{\partial v_\parallel} +\right.$$

$$+ \left.\frac{a\Omega_e v_\parallel}{\Omega v_\perp}\frac{\partial}{\partial v_\perp}\right\}\frac{1}{-\omega + k_\parallel v_\parallel + i0}\frac{\partial}{\partial v_\parallel} f_{0e}\,d\vec{v}, \tag{22}$$

$$R(\vec{K}-\vec{k}) = 1 + \frac{4\pi e^2(\Omega-\omega)}{m\{(\Omega-\omega)^2 - c^2K_\perp^2\}}\sum_{a=-\infty}^{\infty}\int v_\parallel \frac{J_a^2(K_\perp v_\perp/\Omega_e)}{\Omega-\omega+k_\parallel v_\parallel - a\Omega_e}\times$$
$$\times\left\{\left(1-\frac{a\Omega_e}{\Omega-\omega}\right)\frac{\partial}{\partial v_\parallel} + \frac{a\Omega_e v_\parallel}{(\Omega-\omega)v_\perp}\frac{\partial}{\partial v_\perp}\right\} f_{0e}\,d\vec{v}. \tag{23}$$

We now write

$$f_h = f_{h1} + f_{h2}, \tag{24}$$

where f_{h1} corresponds to contribution from the first term (direct coupling) and f_{h2} that from the second term (polarization coupling) in the RHS of Eq. (16). Then we obtain

$$\gamma_1(\vec{K},\Omega) = -\frac{4\pi e f_{h1}}{m\Omega^2}\left(\frac{\partial\varepsilon_0}{\partial\Omega}\right)^{-1}, \tag{25a}$$

$$\gamma_2(\vec{K},\Omega) = -\frac{4\pi e f_{h2}}{m\Omega^2}\left(\frac{\partial\varepsilon_0}{\partial\Omega}\right)^{-1}, \tag{25b}$$

where

$$\gamma(\vec{K},\Omega) = \gamma_1(\vec{K},\Omega) + \gamma_2(\vec{K},\Omega). \tag{25c}$$

Noting that the contribution of P towards the plasma-maser interaction is zero, the growth rate of the O-mode from the first term of Eq.(16) reduces to

$$\frac{\gamma_1(\vec{K},\Omega)}{\Omega} = \sum_{\vec{k},\omega}\delta\frac{\pi^{1/2}}{2}\frac{\omega_{pe}^2}{\Omega^3}\frac{|E_\ell(k,\omega)|^2}{4\pi n_b T_b}\frac{K_\perp^2 v_{em}^2}{|k_\parallel|}\frac{\omega}{k_\parallel}\left(\frac{k_\parallel v_0 - \omega}{k_\parallel v_{em}}\right)\times$$
$$\times\frac{(\Omega^2-\Omega_e^2)\exp[-m(\omega/k_\parallel - v_0)^2/2T]}{\{\Omega_e^2-(\Omega-\omega)^2\}}\left[\frac{\{\Omega(\Omega-\omega)-\Omega_e^2\}}{\Omega_e^2}\times\right.$$
$$\left.\times\left(\frac{k_\parallel^2}{\omega}Y + 2k_\parallel^2 Z\right) + \frac{m}{T}\left\{1+\frac{2(\Omega-\omega)\omega}{\Omega_e^2-(\Omega-\omega)^2}\right\}\right]. \tag{26}$$

where $v_{em} = \left(\frac{2T}{m}\right)^{1/2}$, $Y = \frac{2(\Omega-\omega)}{\Omega_e^2-(\Omega-\omega)^2} + \frac{\Omega}{\Omega(\Omega-\omega)-2\Omega_e^2}$, and

$$Z = \frac{2\Omega(\Omega-\omega)}{\{\Omega_e^2-(\Omega-\omega)^2\}\{\Omega(\Omega-\omega)-\Omega_e^2\}}. \tag{26a}$$

In deriving the above equations, we have taken

$$f_{0e} = (1-\delta)\left(\frac{m}{2\pi T_b}\right)^{3/2} \exp\left[-\frac{mv^2}{2T_b}\right] + \delta\left(\frac{m}{2\pi T}\right)^{3/2} \times$$
$$\times \exp\left[-\frac{mv_\perp^2}{2T}\right] \exp\left[-\frac{m(v_\parallel - v_0)^2}{2T}\right]. \quad (27)$$

In the above, T_b and T are the temperatures of the background and the beam plasmas, respectively, and $n_0/n_b = \delta \ll 1$ is the ratio of the beam to the background plasma density. Also we have used the expression for $(\partial\varepsilon_0/\partial\Omega)$ as obtained from the Vlasov formulation [16] which is valid for $K_\perp v_{eb}/\Omega_e \ll 1$,

$$\frac{\partial\varepsilon_0}{\partial\Omega} \approx \left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \frac{K_\perp^2 v_{eb}^2 \Omega}{(\Omega^2 - \Omega_e^2)^2}, \quad (28)$$

where $v_{eb} = (2T_b/m)^{1/2}$. As the nonlinear force analysis assumes a single particle approximation which does not include the finite temperature effect, we have used the linear dispersion relation [16] as

$$\varepsilon_0(\vec{K}, \Omega) = 1 - \left(\frac{cK_\perp}{\Omega}\right)^2 - \frac{\omega_{pe}^2 K_\perp^2 v_{eb}^2}{2\Omega_e^2(\Omega^2 - \Omega_e^2)}. \quad (29)$$

The growth rate of the O–mode from the second term of Eq.(16) reduces to

$$\frac{\gamma_2(\vec{K}, \Omega)}{\Omega} = \sum_{k,\omega} 2\pi^{1/2}\delta \frac{\omega_{pe}^4 |E_\ell(k,\omega)|^2}{\Omega^5 4\pi n_b T_b} \frac{K_\perp^2 \omega\{(\omega - k_\parallel v_0/k_\parallel v_{em})\}}{k_\parallel |k_\parallel|} \times$$
$$\times \frac{\Omega_e^2(\Omega^2 - \Omega_e^2)}{\{\Omega_e^2 - (\Omega - \omega)^2\}^2} \exp[-m(\omega/k_\parallel - v_0)^2/2T]. \quad (30)$$

In deriving Eqs.(26) and (30), we have retained terms of the Bessel functions $J_n(x)$ etc. for $n = \pm 1$, and also taken $J_1(x) \approx x/2$. Furthermore, the electron plasma frequency is defined for the background plasma as $\omega_{pe} = (4\pi n_b e^2/m)^{1/2}$. The total growth rate of the O–mode in the presence of Langmuir turbulence is given by the sum of Eqs.(26) and (30).

4. EXTRAORDINARY MODE INSTABILITY

We now consider that the test electromagnetic wave is in the extraordinary mode with wave vector $\vec{K} = (K_\perp, 0, 0)$, propagating in the x-direction, and with electric field in the xy–plane.

The linearized electron equation of motion together with Maxwell equations, ignoring plasma temperature and ion motion, and with the nonlinear force term are

$$mn_b \frac{\partial \vec{v}_h}{\partial t} = -en_b \delta \vec{E}_h - \frac{n_b e}{c} \vec{v}_h \times \vec{B}_0 + \vec{F}_{Nh}, \tag{31}$$

$$\nabla \times \delta \vec{E}_h = -\frac{1}{c} \frac{\partial \delta \vec{B}_h}{\partial t}, \tag{32}$$

$$\nabla \times \delta \vec{B}_h = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial}{\partial t} \delta \vec{E}_h, \tag{33}$$

$$\vec{J} = -en_b \vec{v}_h. \tag{34}$$

We take the following quantities as [17]

$$\delta \cdot \vec{E}_h = \hat{x} \delta E_{hx} + \hat{y} \delta E_{hy}, \tag{35}$$

$$\vec{F}_{Nh} = \hat{x} F_{Nhx} + \hat{y} F_{Nhy}, \tag{36}$$

$$F_{Nhx} = \mathrm{i} \delta E_{hx} f_{hx1} + \mathrm{i} \delta E_{hy} f_{hx2}, \tag{37}$$

$$F_{Nhy} = \mathrm{i} \delta E_{hx} f_{hy1} + \mathrm{i} \delta E_{hy} f_{hy2}, \tag{38}$$

From Eqs.(32)–(34), we obtain

$$v_x = -\frac{\mathrm{i} \Omega \delta E_{hx}(\vec{K}, \Omega)}{4\pi e n_b}, \tag{39}$$

$$v_y = \left(\frac{\mathrm{i} K_\perp^2 c^2}{4\pi e n_b \Omega} - \frac{\mathrm{i} \Omega}{4\pi e n_b} \right) \delta E_{hy}(\vec{K}, \Omega). \tag{40}$$

Here, $(\vec{K}, \Omega)$ are the wave vector and the frequency of the X–mode.

Eliminating v_x and v_y, we finally obtain the dispersion relation of the X–mode in the presence of the high–frequency nonlinear force as

$$\left(1 - \frac{\Omega^2}{\omega_{pe}^2}\right)\left(1 - \frac{\Omega^2}{\omega_{pe}^2} + \frac{K_\perp^2 c^2}{\omega_{pe}^2}\right) + \left(\frac{\Omega \Omega_e}{\omega_{pe}^2}\right)\left(\frac{K_\perp^2 c^2 \Omega_e}{\omega_{pe}^2} - \frac{\Omega \Omega_e}{\omega_{pe}^2}\right) -$$
$$-\left(1 - \frac{\Omega^2}{\omega_{pe}^2}\right) \frac{\mathrm{i} f_{hy2}}{en_b} - \left(1 - \frac{\Omega^2}{\omega_{pe}^2} + \frac{K_\perp^2 c^2}{\omega_{pe}^2}\right) \frac{\mathrm{i} f_{hx1}}{en_b} + \frac{\Omega \Omega_e}{en_b} \frac{f_{hx2}}{en_b} -$$
$$-\left(\frac{K_\perp^2 c^2 \Omega_e}{\omega_{pe}^2 \Omega} - \frac{\Omega \Omega_e}{\omega_{pe}^2}\right) \frac{f_{hy1}}{en_b} - \frac{f_{hx1} f_{hy2}}{e^2 n_b} + \frac{f_{hy1} f_{hx2}}{e^2 n_b} = 0. \tag{41}$$

For $\omega_{pe}/cK_\perp \ll 1$, the dispersion relation of the X-mode reduces to, retaining the contribution of the f_{hy2} in the above equation

$$\left(1-\frac{\Omega^2}{\omega_{pe}^2}\right)\left(1-\frac{\Omega^2}{\omega_{pe}^2}+\frac{K_\perp^2c^2}{\omega_{pe}^2}\right)+\frac{\Omega\Omega_e}{\omega_{pe}^2}\left(\frac{K_\perp^2c^2\Omega_e}{\omega_{pe}^2\Omega}-\frac{\Omega\Omega_e}{\omega_{pe}^2}\right)=$$
$$=\left(1-\frac{\Omega^2}{\omega_{pe}^2}\right)\frac{\mathrm{i}f_{hy2}}{en_b}. \quad (42)$$

The above equation can also be expressed as

$$\left(K_\perp^2-\frac{\Omega^2}{c^2}+\frac{\omega_{pe}^2}{c^2}\right)+\left(1-\frac{\Omega^2}{\omega_{pe}^2}\right)^{-1}\frac{\Omega\Omega_e}{c^2}\left(\frac{K_\perp^2c^2\Omega_e}{\omega_{pe}^2\Omega}-\frac{\Omega\Omega_e}{\omega_{pe}^2}\right)=\frac{\mathrm{i}\omega_{pe}^2f_{hy2}}{en_bc^2}. \quad (43)$$

We can write the LHS of Eq.(43) as the linear dielectric function $\varepsilon_0(\vec{K},\,\Omega)$ of the X-mode and get

$$\varepsilon_0(\vec{K},\,\Omega)=\frac{\mathrm{i}\omega_{pe}^2f_{hy2}}{en_bc^2}. \quad (44)$$

Taking $\Omega=\Omega_r+\mathrm{i}\gamma$, where Ω_r is the real frequency of X-mode and γ its growth rate, we get

$$\varepsilon_0(\vec{K},\,\Omega_r)=0, \quad (45)$$

and

$$\gamma=\frac{\omega_{pe}^2f_{hy2}}{en_bc^2}\left(\frac{\partial\varepsilon_0}{\partial\Omega}\right)^{-1}. \quad (46)$$

Substituting for Δf [18] in Eq. (9) and using Eq. (38), after a lengthy but straight forward calculations, we obtain the nonlinear force (f_{hy2}). Substituting for the values of f_{hy2} and $\partial\varepsilon_0/\partial\Omega$ in Eq. (46), we finally obtain the growth rate of X-mode in the presence of Langmuir turbulence due to the high-frequency nonlinear force as

$$\frac{\gamma}{\Omega} = -2\left(\frac{e}{m}\right)^2\omega_{pe}^2(\Omega_e-\Omega)^2\Omega_e^{-1}\sum_{k,\omega}\frac{(\Omega-\omega)|E_\ell(k,\omega)|^2}{\{(\Omega-\omega)^2-c^2(K_\perp^2+k_\parallel^2)\}S'}\times$$
$$\times\ I_m[(A'+B')\times(C'+D')], \quad (47)$$

where

$$A' = \sum_{s,t,a,b=-\infty}^{\infty}\int\frac{v_yJ_s(K_\perp v_\perp/\Omega_e)J_t(K_\perp v_\perp/\Omega_e)\exp[\mathrm{i}(s-t)\phi]}{\mathrm{i}(t\Omega_e-\Omega)}\times$$
$$\times\ \frac{\partial}{\partial v_\parallel}\frac{J_a(K_\perp v_\perp/\Omega_e)\exp[\mathrm{i}(a-b)\phi]}{b\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}\frac{(J_{b+1}-J_{b-1})}{2}\times$$

$$\times \left\{\left(1+\frac{k_\parallel v_\parallel}{\Omega-\omega}\right)\frac{\partial}{\partial v_\perp}-\frac{k_\parallel v_\perp}{\Omega-\omega}\frac{\partial}{\partial v_\parallel}\right\} f_{0e}\,d\vec{v}, \tag{48}$$

$$B' = \sum_{s,t=-\infty}^{\infty}\int \frac{v_y J_s(K_\perp v_\perp/\Omega_e)\exp[i(s-t)\phi]}{i(t\Omega_e-\Omega)}\frac{(J_{t+1}-J_{t-1})}{2}\times$$
$$\times \left\{\left(1+\frac{k_\parallel v_\parallel}{\Omega-\omega}\right)\frac{\partial}{\partial v_\perp}-\frac{k_\parallel v_\perp}{\Omega-\omega}\frac{\partial}{\partial v_\parallel}\right\}\times$$
$$\times \frac{1}{-(\omega-k_\parallel v_\parallel+i0)}\frac{\partial}{\partial v_\parallel} f_{0e}\,d\vec{v}, \tag{49}$$

$$C' = A'[\vec{K}-\vec{k}\leftrightarrow\vec{K},\ \Omega-\omega\leftrightarrow\Omega,\ \omega\to-\omega,\ \vec{k}\to-\vec{k}], \tag{48a}$$

$$D' = B'[\vec{K}-\vec{k}\leftrightarrow\vec{K},\ \Omega-\omega\leftrightarrow\Omega,\ \omega\to-\omega,\ \vec{k}\to-\vec{k}], \tag{49a}$$

$$S' = 1-\frac{\omega_{pe}^2(\Omega-\omega)}{(\Omega-\omega)^2-c^2(K_\perp^2+k_\parallel^2)}\sum_{a,b=-\infty}^{\infty}\times$$
$$\times \int \frac{v_y J_a(K_\perp v_\perp/\Omega_e)\exp[i(a-b)\phi]}{b\Omega_e-k_\parallel v_\parallel-(\Omega-\omega)}\frac{(J_{b+1}-J_{b-1})}{2i}\times$$
$$\times \left[\left(1+\frac{k_\parallel v_\parallel}{\Omega-\omega}\right)\frac{\partial}{\partial v_\perp}-\frac{k_\parallel v_\perp}{\Omega-\omega}\frac{\partial}{\partial v_\parallel}\right] f_{0e}\,d\vec{v}. \tag{50}$$

In deriving the above equation, we have used the relation $\partial\varepsilon_0/\partial\Omega \approx \approx -\omega_{pe}^2\Omega_e/2c^2(\Omega_e-\Omega)^2$ as obtained from the Vlasov formulation [18]. The results for O–mode [Eqs.(26) and (30)] and X–mode [Eq.(47)] from the nonlinear force consideration are identical with those obtained from direct approach following Vlasov formulation [16, 18].

5. SOLAR RADIO EMISSION

It is generally agreed that Langmuir turbulence is related to the emission mechanism of solar flares [19]. There exist several explanation of solar microwave bursts such as the transformation of Langmuir waves to electromagnetic waves [20], and cyclotron maser [21]. The relative role of there two types of mechanisms depends on the ratio ω_{pe}/Ω_e, where ω_{pe} is the Langmuir frequency and Ω_e is the electron–cyclotron frequency. If $\omega_{pe}/\Omega_e \gg 1$, then the transformation mechanism is important, however for $\omega_{pe}/\Omega_e \ll 1$, the cyclotron maser gives the dominant emission mechanism. In the intermediate

case $\omega_{pe}/\Omega_e \lesssim 1$, which is typical for solar flares, the plasma-maser mechanism plays an important role which is studied here. We estimate the growth rates for O– and X-mode under the condition $\omega_{pe} \lesssim \Omega_e$ [22].

A. Growth rate of ordinary mode

The linear dispersion curves [Eq.(29)] for $\Omega_e > \omega_{pe}$ are shown by solid lines in Fig.1.

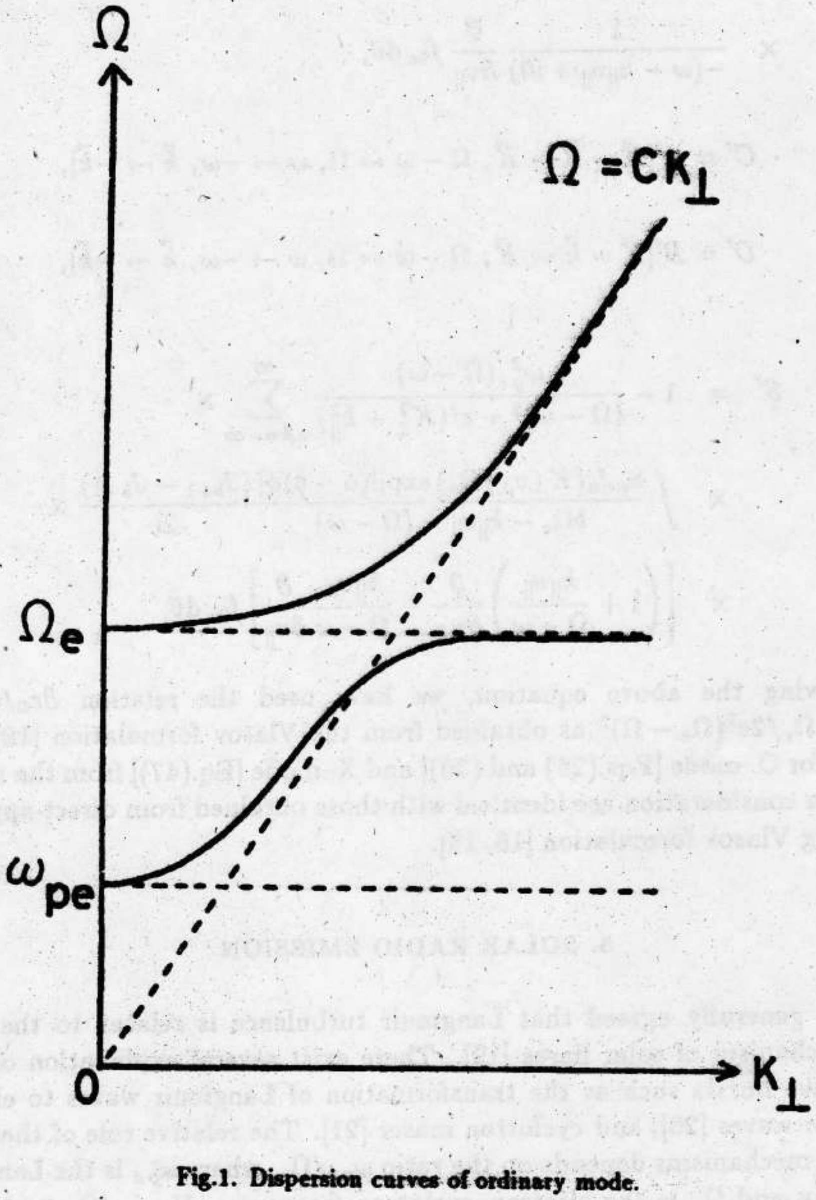

Fig.1. Dispersion curves of ordinary mode.

The dispersion relation is solved near $\Omega = \Omega_e$ by setting $\Omega = = \Omega_e(1 + \Delta)$, we

obtain

$$\Delta = \frac{\omega_{pe}^2 (K_\perp v_{eb})^2}{4\Omega_e^2(\Omega_e^2 - c^2 K_\perp^2)}. \tag{51}$$

Inserting Eq.(51) into Eq.(26), we find that the dominant contribution comes from the last term in the RHS of Eq.(26). Thus the direct coupling term gives

$$\frac{\gamma_1}{\Omega_e} = \sum_{k,\omega} \frac{\delta\pi^{1/2}}{2\Delta} \frac{\omega_{pe}^2 \omega^2}{\Omega_e^4} \frac{|E_\ell(k,\omega)|^2}{4\pi n_b T_b} \frac{K_\perp^2 V_{em}^2}{k_\| |k_\||} \times$$
$$\times \frac{m}{T}\left(\frac{k_\| v_0 - \omega}{k_\| V_{em}}\right) \exp\left[-\frac{m(\omega/k_\| - v_0)^2}{2T}\right]. \tag{52}$$

Then, for $v_0 > \omega/k_\|$, the ordinary mode grows ($\gamma_1 > 0$) if $\Delta > 0$ which means $\Omega_e > cK_\perp$.

Inserting Eq.(51) into Eq.(30), we get the growth rate from the polarization term

$$\frac{\gamma_2}{\Omega_e} = \sum_{k,\omega} \frac{\delta\pi^{1/2}}{\Delta} \frac{\omega_{pe}^4 \omega}{\Omega_e^5} \frac{|E_\ell(k,\omega)|^2}{4\pi n_b T_b} \left(\frac{K_\perp}{k_\|}\right)^2 \times$$
$$\times \left(\frac{\omega - k_\| v_0}{k_\| V_{em}}\right) \exp\left[-\frac{m(\omega/k_\| - v_0)^2}{2T}\right]. \tag{53}$$

The polarization term gives growth ($\gamma_2 > 0$) for $\Delta < 0$ with $v_0 > \omega/k_\|$.

The ratio of both contributions ($R = \gamma_1/|\gamma_2|$) reduces to

$$R = \frac{\Omega_e}{2\omega_{pe}}. \tag{54}$$

Accordingly, for $\Omega_e > 2\omega_{pe}$ which is typical for solar conditions, the dominant plasma–maser contribution comes from the direct coupling term. The ordinary mode becomes unstable ($\gamma_1 > 0$) just above the electron cyclotron frequency ($\Delta > 0$) for $v_0 > \omega/k_\|$. On the other hand, the ordinary mode damps ($\gamma_1 < 0$) below the electron cyclotron frequency ($\Delta < 0$). If we assume for some typical solar flare parameters that $\delta = n_0/n_b = 10^{-4}$, $T/T_b = 10^3$, $\omega/k_\| = 10 v_{em}$, $v_0 = 11 v_{em}$, then

$$\frac{\gamma_1}{\Omega_e} = \sum_{k,\omega} 2\pi^{1/2}\delta \left(\frac{\omega_{pe}}{k_\| v_{eb}}\right)^2 \frac{|E_\ell(k,\omega)|^2}{4\pi n_b T_b} \left(\frac{k_\| v_0 - \omega}{k_\| v_{em}}\right) \times$$
$$\times \exp\left[-\frac{m(\omega/k_\| - v_0)^2}{2T}\right] \approx 10^{+1} \sum_{k,\omega} \frac{|E_\ell(k,\omega)|^2}{4\pi n_b T_b}. \tag{55}$$

In deriving Eq.(55), we put $\Delta \approx (\omega_{pe} K_\perp v_{eb}/2\Omega_e^2)^2$. Thus, we find that the growth rate [Eq.(55)] obtained for solar conditions is large enough to play an important role.

B. Growth rate of extraordinary mode

The linear dispersion relation of X–mode reduces to [18]

$$K_\perp^2 - \frac{\Omega^2}{c^2} + \frac{\omega_{pe}^2 \Omega^2}{c^2(\Omega^2 - \Omega_e^2)} = 0. \quad (56)$$

Setting $\Omega = \Omega_e(1 + \Delta)$, we get

$$\Delta = \frac{\omega_{pe}^2}{2(\Omega^2 - c^2 K_\perp^2)}. \quad (57)$$

Inserting Eq.(57) into Eq.(47), we obtain

$$\frac{\gamma}{\Omega_e} = \frac{\pi^{1/2}\delta}{8\Delta^2}\left(\frac{\omega_{pe}}{\Omega_e}\right)^4 \left(\frac{K_\perp v_{eb}}{\Omega_e}\right)^2 \sum_{k,\omega} \frac{|E_\ell(k,\omega)|^2}{4\pi n_b T} \times$$

$$\times \frac{\Omega_e}{\omega}\frac{k_\|}{|k_\||}\left(\frac{\omega - k_\| v_0}{k_\| v_{em}}\right) \exp\left[-\frac{m(\omega/k_\| - v_0)^2}{2T}\right]. \quad (58)$$

Thus, we find that extraordinary mode grows for $v_0 < \omega/k_\|$.

Next, the radio of growth rates for both modes $[Q = \gamma_1/\gamma]$ reduces to

$$Q = \frac{4T}{T_b}\left(\frac{\omega_{pe}}{k_\| v_{eb}}\right)^2 \frac{\Omega_e \omega_{pe}}{K_\perp^2 v_{eb}^2} \gg 1. \quad (59)$$

Accordingly, we may conclude that the emission in the ordinary mode is dominant for solar flare conditions.

6. DISCUSSIONS

As has already been mentioned, the plasma–maser interaction can be understood on the basis of a high–frequency dissipative nonlinear force [15]. This nonlinear force arises as a result of the resonant interaction between electrons and modulated fields caused by coupling between the test wave field and the turbulence field in the system. Eqs.(15) and (46) have shown that the O–mode and X–mode instabilities as considered in the present work are made unstable by the high–frequency nonlinear force.

This concept of dissipative nonlinear force has already been used by the present authors to show that a high–frequency electromagnetic instability is caused by this dissipative nonlinear force in an unmagnetized plasma in the presence of Langmuir turbulence driven by an electron beam [15]. The principal purpose of the present work is to extend the earlier work to the electromagnetic emission in a magnetized plasma. We have considered here the two familiar cases of the O–mode and X–mode emission. It has been seen that the electromagnetic emission in the O–mode and X–mode are also made unstable by the dissipative nonlinear force discussed here.

The results obtained from the nonlinear force consideration are identical with those obtained from direct approach following Vlasov formulation [16, 18]. This similarity in the results strengthens the concept of the dissipative nonlinear force which acts as the force driving the instability.

Finally, the results are applied to the emission mechanism of solar flare. For $2\omega_{pe} \lesssim \Omega_e$, which is typical for solar flare condition, the dominant electromagnetic emission comes from the O–mode [Eq.(55)]. It is found that the ordinaly mode emission occurs just above the electron cyclotron frequency ($\Omega \gtrsim \Omega_e$) for $v_0 > \omega/k_{\|}$. We may conclude that there are three basically different emission mechanisms for solar microwave bursts. The first mechanism is the transformation of Langmuir waves to electromagnetic waves [20] which is effective for $\omega_{pe} \gg \Omega_e$. The second one is the cyclotron maser [21] which is important for $\omega_{pe} \ll \Omega_e$. On the other hand, the third mechanism is plasma–maser effect considered here, which is effective for $\omega_{pe} \lesssim \Omega_e$. Furthermore, the plasma–maser process studied here may play a potential importance to interpret numerous enhanced electromagnetic radiation from the Langmuir turbulence observed in the laboratory [23–25] and astrophysical plasmas [19, 26].

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